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Department of Mechanical Engineering

Graduation Project

Design and Analysis of Dependent Suspensions
Systems in Vehicles

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1.Introduction to Suspension Design

Ride quality and handling are two of the most important issues related to vehicle refinement. Together they produce some design conflicts that have to be resolved by

compromise. Also, the wide range of operating conditions experienced by a vehicle This all adds up to some very challenging tasks for the suspension designer. In order to understand the issues facing the suspension designer it is necessary to

have knowledge of:

- The requirements for steering, handling and stability
- The ride requirements related to the isolation of the vehicle body from road irregularities and other sources of vibration and noise
- How tyre forces are generated as a result of braking, accelerating and cornering
- The needs for body attitude control
- Suspension loading and its influence on the size and strength of suspension members.

2.The Role of a Vehicle Suspension

The principal requirements for a vehicle suspension are:

- To provide good ride and handling performance—this requires the suspension to have vertical compliance to provide chassis isolation while ensuring that the wheels follow the road profile with minimum tyre load fluctuation.
- To ensure that steering control is maintained during manoeuvring—this requires the wheels to be maintained in the proper positional attitude with respect to the road surface.
- To ensure that the vehicle responds favourably to control forces produced by the tyres as a result of longitudinal braking and accelerating forces, lateral cornering forces and braking and accelerating torques—this requires the suspension geometry to be designed to resist squat, dive and roll of the vehicle body.
- To provide isolation from high frequency vibration arising from tyre excitation
—this requires appropriate isolation in the suspension joints to prevent the transmission of “road noise” to the vehicle body.

- To provide the structural strength necessary to resist the loads imposed on the suspension.

3.Suspension Classifications

In general suspensions can be broadly classified as dependent, independent or semi-dependent types. With dependent suspensions the motion of a wheel on one side of the vehicle is

dependent on the motion of its partner on the other side. For example, when a wheel on one side of an axle strikes a pothole, the effect of it is transmitted directly to its partner on the other side. Generally this has a detrimental effect on the ride and handling of the vehicle.

As a result of the trend to greater vehicle refinement, dependent suspensions are no longer common on passenger cars. However, they are still commonly used on commercial and off-road vehicles. Their advantages are simple construction and

almost complete elimination of camber change with body roll (resulting in low tyre wear). Dependent suspensions are also commonly used at the rear of front-wheel drive light commercial vehicles and on commercial and off-highway vehicles with

rear driven axles (live axles). They are occasionally used in conjunction with non-driven axles (dead axles) at the front of some commercial vehicles with rear wheel drive.

With independent suspensions the motion of wheel pairs is independent, so that a disturbance at one wheel is not directly transmitted to its partner. This leads to better ride and handling capabilities. This form of suspension usually has benefits in

packaging and gives greater design flexibility when compared to dependent systems. Some of the most common forms of front and rear independent suspension

designs are considered below. McPherson struts, double wishbones and multi-link

systems are commonly employed for both front and rear wheel applications. Trailing arm, semi-trailing arm and swing axle systems tend to be used predominantly for rear wheel applications.

There is also a group of suspensions that fall some way between dependent and independent suspensions and are consequently called semi-dependent. With this form of suspension, the rigid connection between pairs of wheels is replaced by a compliant link. This usually takes the form of a beam that can twist providing both positional control of the wheel carrier as well as compliance. Such systems tend to be simple in construction while having scope for design flexibility when used in conjunction with compliant supporting bushes

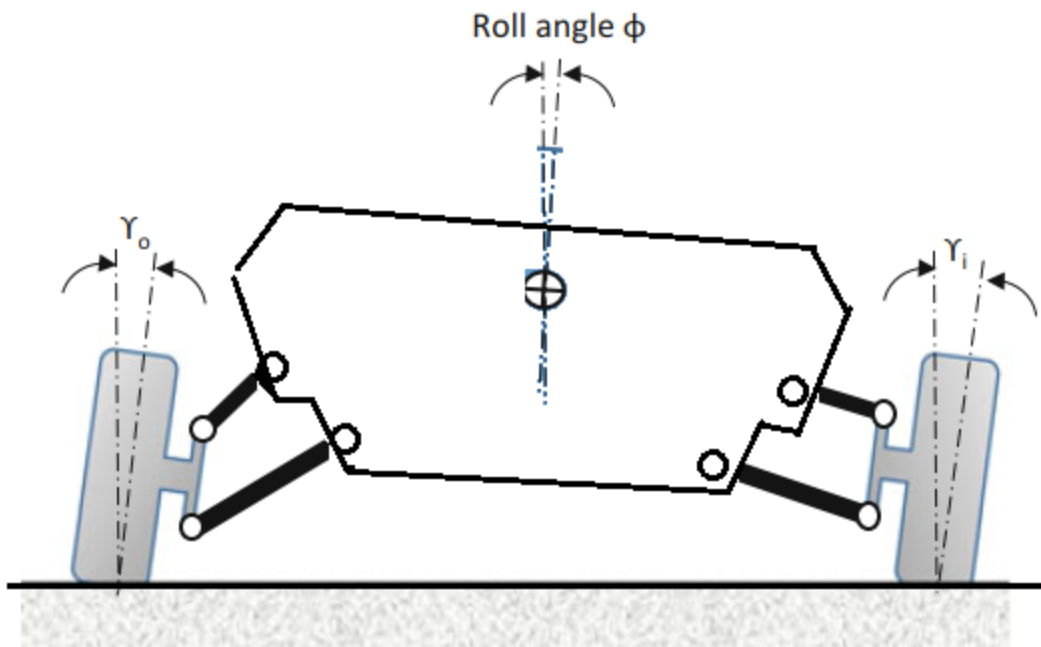
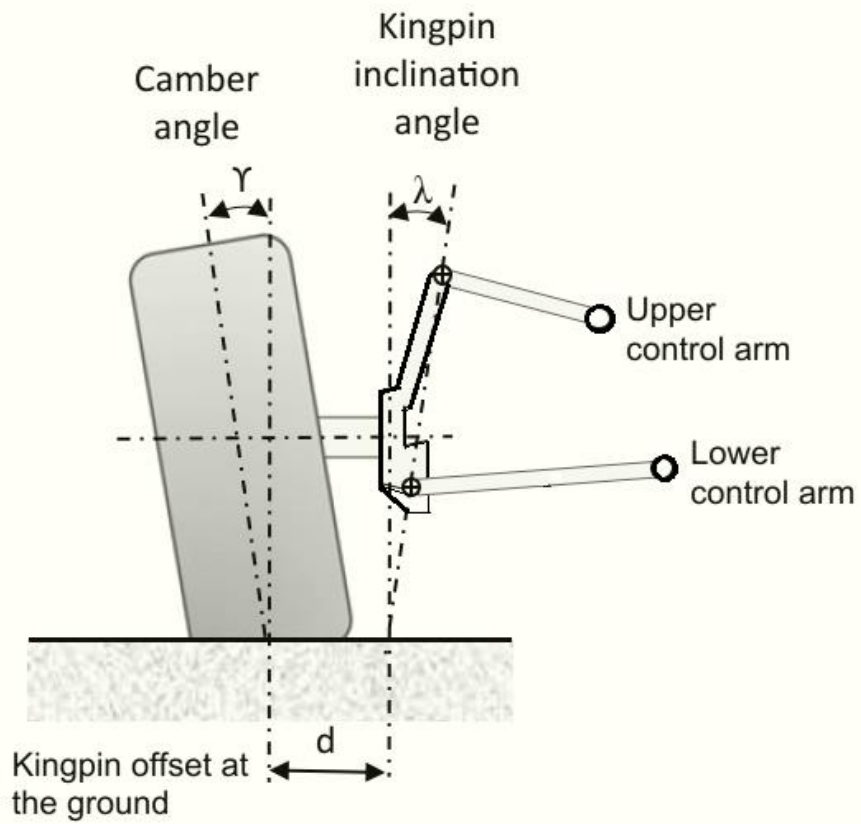
4. Defining Wheel Position

Since one of the most important functions of a suspension system is to control the position of the road wheels, it is important to understand the definitions relating to wheel location. The location of the wheels relative to both the road and the vehicle suspension is important and will in general be affected by suspension deflection and tyre loading. In the following subsections, parameters relating to wheel position are defined and their effect on handling behaviour is considered.

Camber Angle

This is the angle between the wheel plane and the vertical—taken to be positive when the wheel leans outwards from the vehicle

A disadvantage of an independent suspension is that the wheels incline relative to the vehicle body on a bend. This tends to produce increased negative camber on the inner wheels and increased positive camber on the outer wheels



kingpin inclination (KPI)

Is the angle between the kingpin axis and the vertical. It has the effect of causing the vehicle to rise when the wheels are turned and produces a noticeable self-centring effect for KPI's greater than 15°

Kingpin Offset

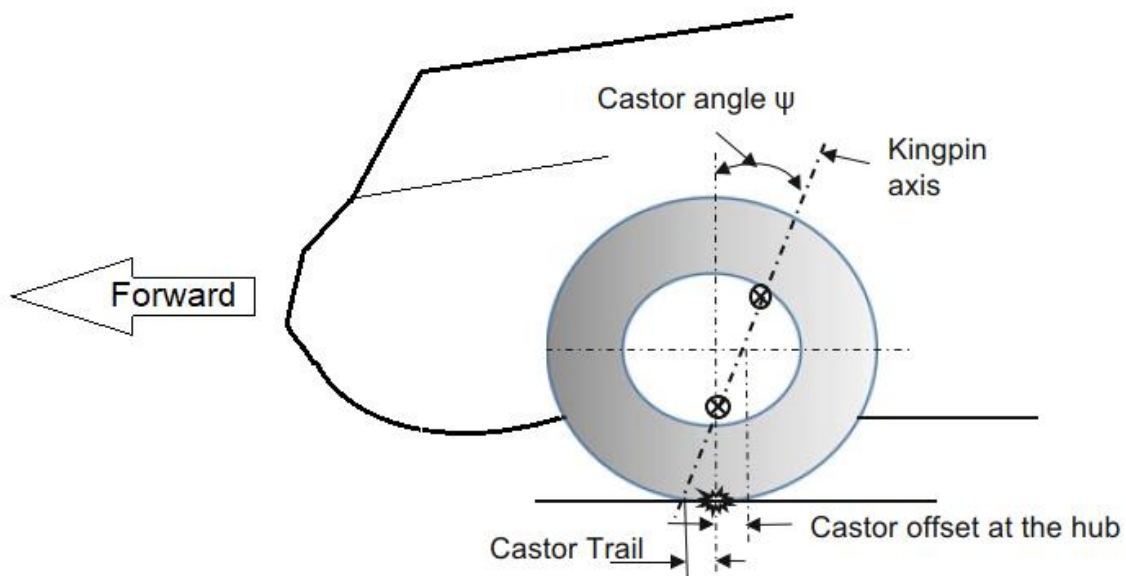
Kingpin offset (KPO) is the distance between the centre of the tyre contact patch and the intersection of the kingpin axis at the ground plane, KPO can be changed by changing tire width. Increasing KPO improves the returnability of the steering

Kingpin Offset at the Hub

This is defined as the horizontal distance from the kingpin axis to the intersection of the hub axis and the tire centerline

Castor Angle

Castor angle produces a self-aligning torque for non-driven wheels



Castor Trail

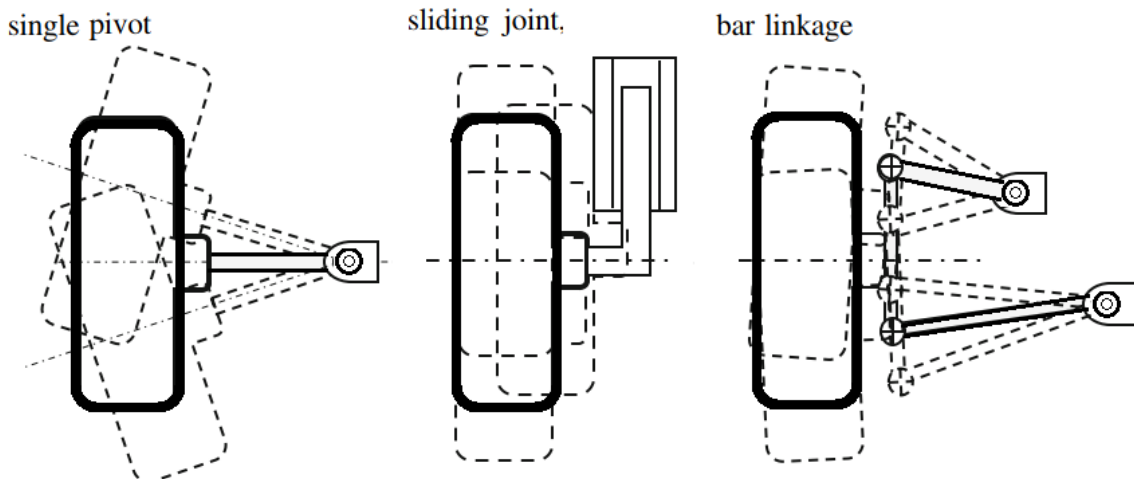
Castor trail, often called mechanical trail, is the longitudinal distance from the point of intersection of the kingpin axis and the ground to the centre of the tyre contact. The effects of castor trail on vehicle behaviour are as follows:

Straight line stability—improved with higher castor trail. Returnability stronger with higher castor trail.

Braking stability is enhanced with higher levels of castor trail.

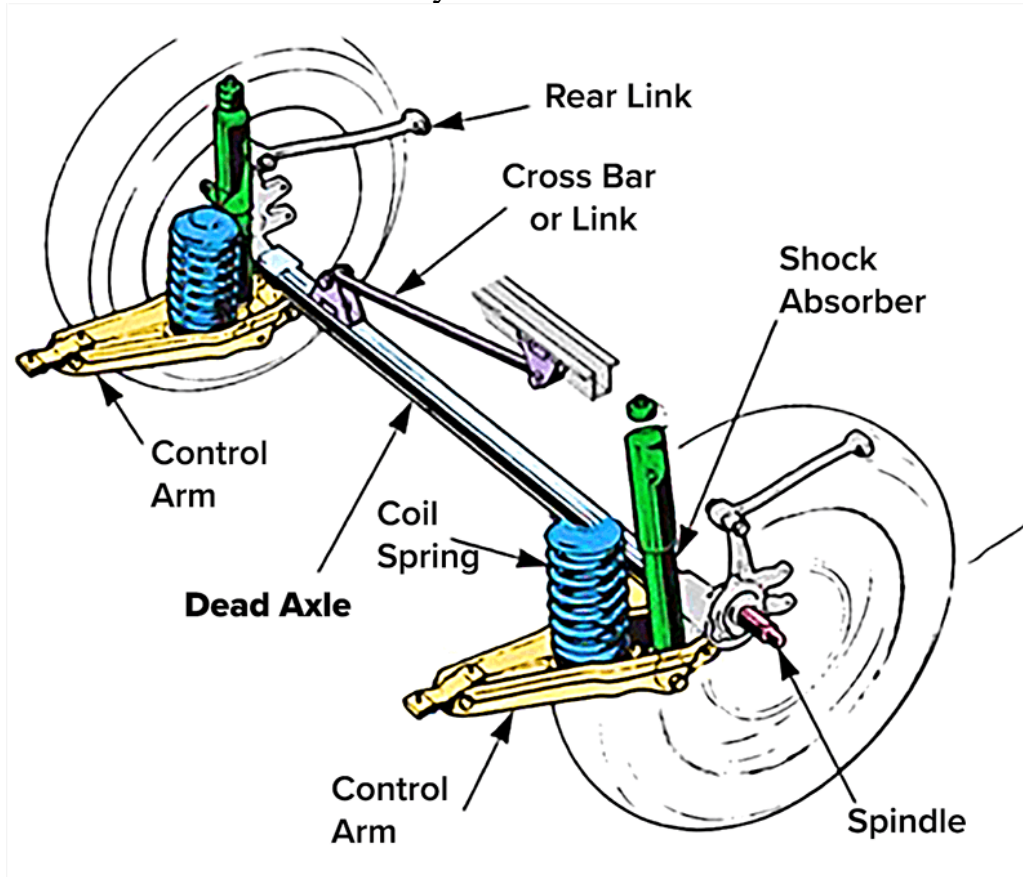
5. Kinematic Requirements for Dependent and Independent Suspensions

The main objective in suspension design is to isolate the vehicle body from road surface undulations. Ideally, this requires that the wheels of the vehicle perfectly follow the road surface undulations with no vertical movement of the vehicle body. In an independent suspension the mechanism coupling the wheel to the body of the vehicle is said to have a single degree of freedom, i.e. the motion of the wheel relative the body can be described by a single coordinate. If it is necessary to couple a pair of wheels together on a single axle (as in a dependent suspension), the mechanism required must have two degrees of freedom to allow vertical movement of each wheel while at the same time having a near rigid connection between the two wheels.



6. Dependent Suspensions

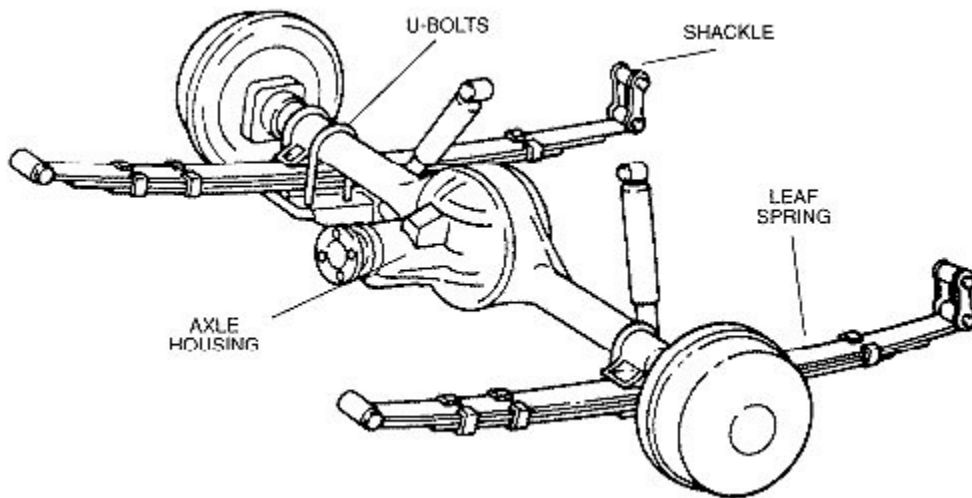
Drive axles: solid axles carry the driven wheels and differential.



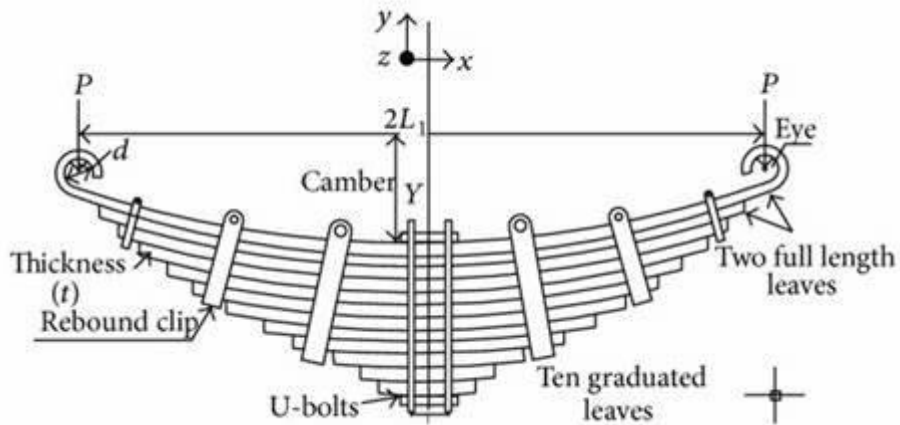
Non-driven axles—: A solid beam simply connects the two none driven wheels which may be steerable.

7. Leaf Springs

It is simple suspension and produces satisfactory location of the wheels with the minimum number of components. Its simplicity derives from the leaf spring properties, i.e. compliant in the vertical direction but relatively stiff laterally and longitudinally.

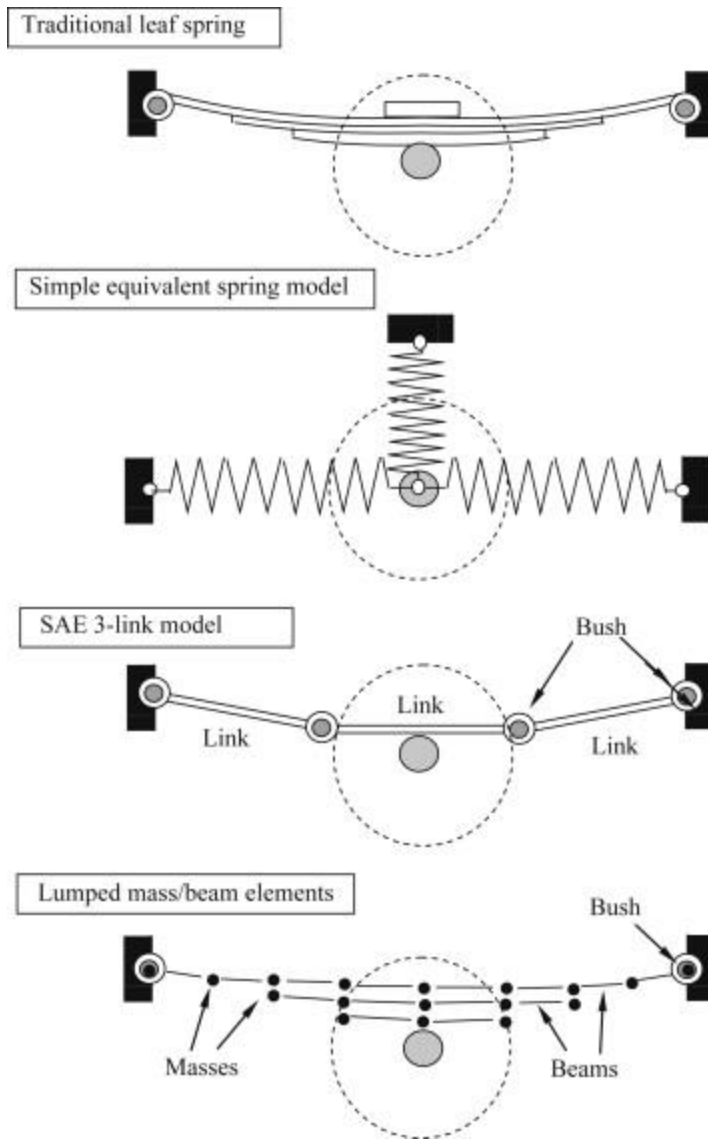


Early cars had steel leaf springs, made up of layers of mild steel strapped together. Some trucks still have leaf springs, On the other hand, most cars have coil springs, which are capable of greater compression than leaf springs. Coil springs, however, vary greatly in their stiffness, Their ease of compression. Naturally, their mechanical behaviour is determined by the properties of the metal alloy used to form the coil – namely, its gauge and intrinsic shear modulus – as well as by the diameter of the coil itself. Strong springs, which compress with difficulty, are fitted to heavy vehicles such as trucks. When the truck is fully laden, the springs are able to support the total vehicle weight over rough ground, but when unladen the weight is insufficient to compress the spring fully and the ride is bouncy. Conversely, vehicles such as luxury cars, buses and taxis that are not intended to go over rough ground have comparatively weak springs in order to give a soft ride. Some cars that are available as ‘sports models’ have stiffer suspensions than their family-oriented counterparts.



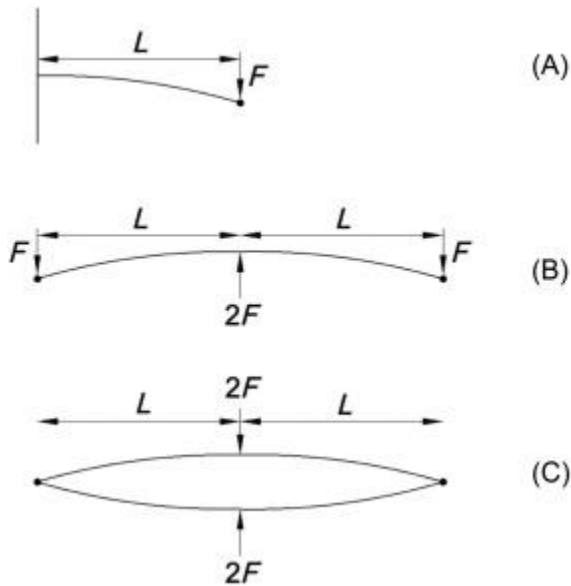
8. Modelling leaf springs

Although the modelling of leaf springs is now rare on passenger cars they are still fitted extensively on light trucks and goods vehicles where they offer the advantage of providing relatively constant rates of stiffness for large variations in load at the axle. The modelling of leaf springs has always been more of a challenge in an MBS environment when compared with the relative simplicity of modelling a coil spring. Several approaches may be adopted,



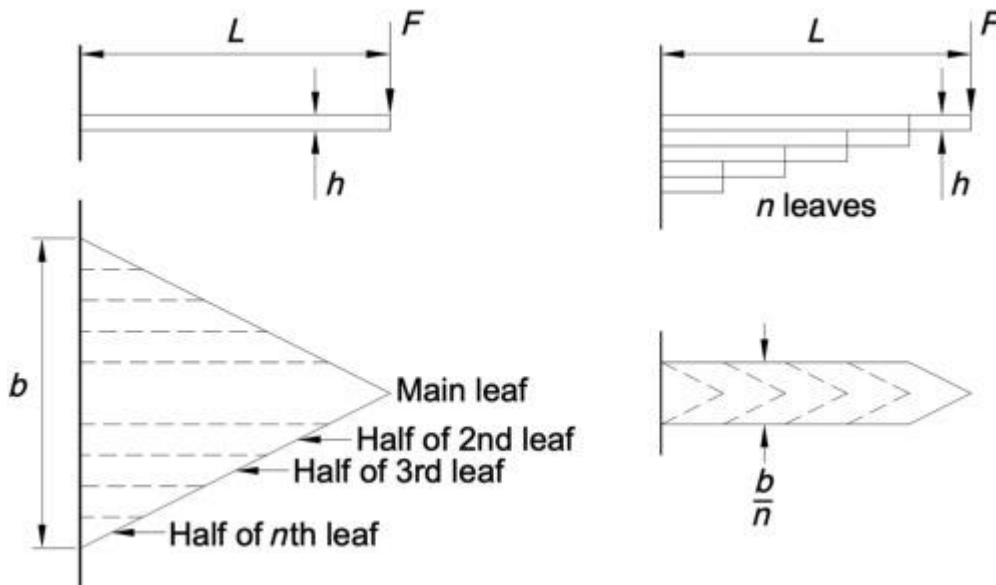
Leaf Springs

Leaf springs consist of one or more flat strips of material loaded as cantilevers or simple beams



The concept used in producing compact cantilever springs of uniform bending stress is to chop the triangular form

The multileaf spring shown and the single triangular section beam both have the same stress and deflection characteristics with the exceptions that the multileaf spring is subject to additional damping due to friction between the leaves and that the multileaf spring can carry a full load in only one direction due to the tendency for the leaves to separate. Leaf separation can be partially overcome by the provision of clips around the leaves.



The deflection of a triangular leaf spring is given by

where F is the force (N), L is the length (m), E is Young's modulus (N/m²), and I is the second moment of area (m⁴).

The deflection of a triangular leaf spring is given by

$$\delta = \frac{FL^3}{2EI}$$

For a rectangular cross section:

$$I = \frac{bh^3}{12}$$

The spring rate is given by

$$k = \frac{F}{\delta} = \frac{Ebh^3}{6L^3}$$

The spring rate is given by

$$k = \frac{F}{\delta} = \frac{Ebh^3}{6L^3}$$

The corresponding bending stress is given by

$$\sigma = \frac{6FL}{bh^2}$$

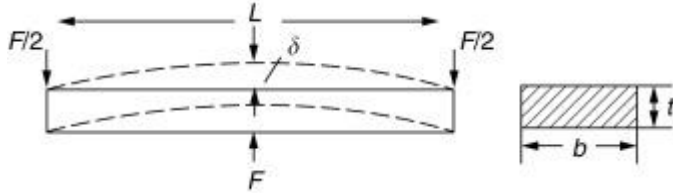
F is the force (N), b is the width (m)

L is the length (m) h is the thickness (m)

E is Young's modulus (N/m²) I is the second moment of area (m⁴)

9. Yield-Limited Design

Even leaf springs can take many different forms, but all of them are basically elastic beams loaded in bending. For the loading

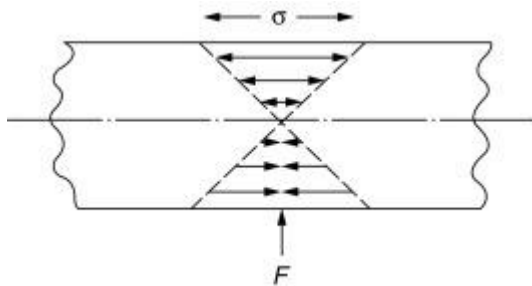


$$\delta = \frac{FL^3}{4Ebt^3}$$

The elastic energy stored in the spring, per unit volume, is

$$U^{\text{el}} = \frac{1}{2} \frac{F\delta}{btL} = \frac{F^2 L^2}{8Eb^2 t^4}$$

the stress in the beam is zero along the neutral axis at its center, and is a maximum at the surface, at the midpoint of the beam (because the bending moment is biggest there).



Now to be successful, a spring must not undergo a permanent set during use: it must always “spring” back. The condition for this is that the maximum stress must always be less than the yield stress:
 Now to be successful, a spring must not undergo a permanent set during use: it must always “spring” back. The condition for this is that the maximum stress must always be less than the yield stress

$$\sigma = \frac{Mc}{I} = \frac{F}{2} \times \frac{L}{2} \times \frac{t}{2} \times \frac{12}{bt^3} = \frac{3FL}{2bt^2}$$

$$\frac{3FL}{2bt^2} < \sigma_y$$

$$U^{e1} = \frac{1}{18} \left(\frac{\sigma_y^2}{E} \right)$$

So if in service a spring has to undergo a given deflection δ under a force F , the ratio of σ_y^2/E must be high enough to avoid a permanent set, The best springs are made of materials with high values of this quantity. For this reason spring materials are heavily strengthened

Case Studies

The translation and the selection

Consider a single-leaf spring first (Figure 10.8(a)). A leaf spring is an elastically bent beam. The energy stored in a bent beam, loaded by a force F , is

$$U = \frac{1}{2} \frac{F^2}{S_B}$$

where S_B , the bending stiffness of the spring

$$S_B = \frac{C_1}{12L^3} E \phi_B^e A^2$$

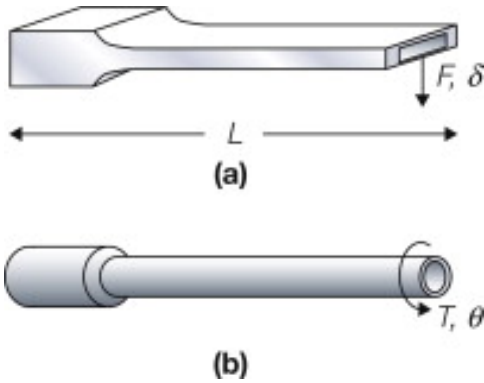


Figure 10.8

Hollow springs use material more efficiently than solid springs (a). Best in bending is the hollow rectangular or elliptical section; best in torsion is the tube (b).

The force F in Equation

$$U = \frac{1}{2} \frac{F^2}{S_B}$$

is limited by the onset of yield; its maximum value is

$$F_f = C_2 Z \frac{\sigma_f}{L} = \frac{C_2}{6L} \sigma_f \phi_B^f A^{3/2}$$

The constants C_1 and C_2 are tabulated and you can get their value from Tables.

Assembling these gives the maximum energy the spring can store:

$$\frac{U_{\max}}{V} = \frac{C_2^2}{6C_1} \left(\frac{(\phi_B^f \sigma_f)^2}{\phi_B^e E} \right)$$

where $V = A L$ is the volume of solid in the spring. The best material and shape for the spring

the one that uses the least material is that with the greatest value of the quantity

$$M_1 = \frac{(\phi_B^f \sigma_f)^2}{\phi_B^e E}$$

For a fixed section shape, the ratio involving the two ϕ s is a constant: The best choice of material is that with the greatest value of

$$\sigma_f^2 / E \text{ —}$$

the same result as before. When shape is a variable, the most efficient shapes are those with large

$$\left(\phi_B^f\right)^2 / \phi_B^e.$$

Values for these ratios are tabulated for common section shapes in Table 10.10 ; hollow-box and elliptical sections are up to three times more efficient than solid shapes.

Section Shape	$(\phi_B^f)^2 \phi_B^e$	$(\phi_T^f)^2 \phi_T^e$
	1	$1.08 \frac{b^2}{h^2} \frac{1}{(1+0.6\frac{b}{h})^2 (1-0.58\frac{b}{h})}$
	0.38	0.83
	0.75	1.6
	0.75	$0.8 \left(1 + \frac{a^2}{b^2}\right) (a \leq t; b)$
	1.5	3.2
	$\frac{(1+3b/h)}{(1+b/h)} (h, b \geq t; t)$	$3.32 \frac{1}{(1-t/h)^4} \dots\dots\dots (h, b \geq t; t)$
	$\frac{3}{4} \frac{(1+3b/a)}{(1+b/a)} (a, b \geq t; t)$	$3.2 \frac{(1+a^2/b^2)}{(1+a/b)} \left(\frac{b}{a}\right)^{3/2} (a, b \geq t; t)$
	3	-
	$\frac{(1+3b/h)}{(1+b/h)} (h, b \geq t; t)$	$1.07 \frac{(1+4h/b)}{(1+h/b)} (h, b \geq t; t)$
	$1.13 \frac{(1+4bt^2/h^3)}{(1+b/h)} (h \geq t; t)$	$0.54 \frac{(1+8b/h)}{(1+b/h)} (h, b \geq t; t)$
	$1.13 \frac{(1+4bt^2/h^3)}{(1+b/h)} (h \geq t; t)$	$1.07 \frac{(1+4h/b)}{(1+h/b)} (h, b \geq t; t)$

Table 10

10.Stability and Handling Comparison of Single Taper-Leaf and Multi-Leaf Spring Suspensions

The handling stability is one of the extremely important performances that affect the driving safety of trucks. How to obtain the strong handling stability for improving the driving safety is an important issue in vehicle design

the lightweight design of trucks is of great importance to enhance the load capacity and reduce the production cost, For leaf springs, the lightweight design of trucks is mainly reflected in replacing the multi-leaf spring with the taper-leaf spring

11.Modeling of the Taper-Leaf and the Multi-Leaf Spring

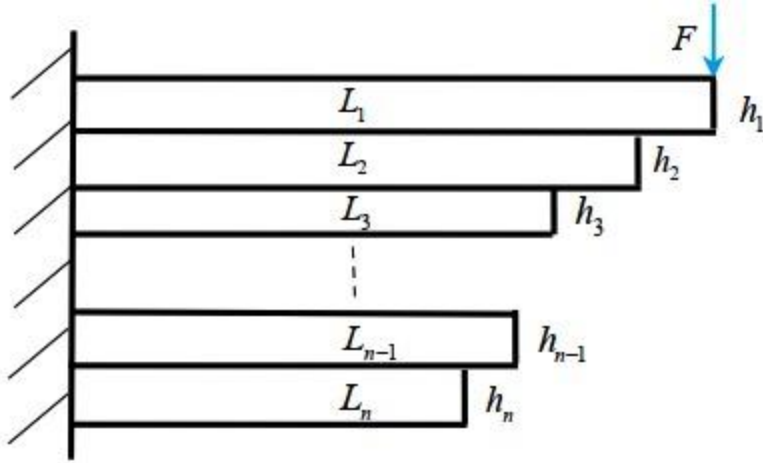
The Mechanical Model of the Multi-Leaf Spring

. The half of the multi-leaf spring can be regarded as an elastic beam. It is fixed at one end. A concentrated load F is applied at the other end. It is assumed that

under F , each piece does not separate from each other and two adjacent pieces have the same

deflection at the contact point. w is the end displacement of the multi-leaf spring. n is the number of slices for the multi-leaf spring. b is the width of each piece. The length parameters of half of the

spring pieces are $L_1, L_2, L_3, \dots, L_{n-1}, L_n$ from large to small respectively. Correspondingly, the thickness parameters of the half of the spring pieces are $h_1, h_2, h_3, \dots, h_{n-1}, h_n$, and E is the elastic modulus



The mechanical model of the multi-leaf spring.

Fig(A-1)

Based on the mechanical model in Fig(A-1) , the stiffness of the half of the multi-leaf spring can be expressed as

$$K = \frac{bE}{4} \left[\sum_{i=2}^n \frac{(L_1 - L_i)^3 - (L_1 - L_{i-1})^3}{h_1^3 + h_2^3 + \dots + h_{i-1}^3} + \frac{L_1^3 - (L_1 - L_n)^3}{h_1^3 + h_2^3 + \dots + h_{n-1}^3 + h_n^3} \right],$$

12.The Mechanical Model of the Single Taper-Leaf Spring

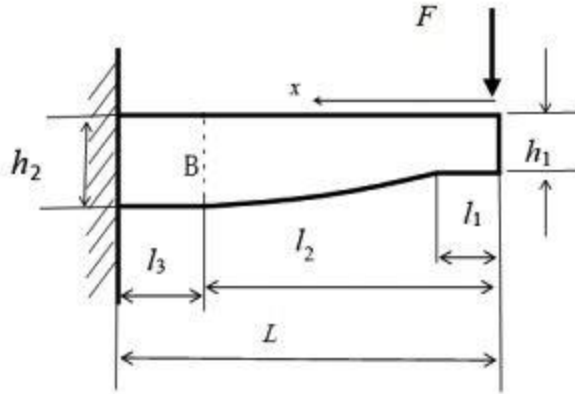
The half of the single taper-leaf spring also can be regarded as an elastic beam. It is fixed at one end. A concentrated load F is applied at the other end.

The geometric

parameters and the coordinate system are marked. L is the half length of the single taper-leaf

spring; h_2 is the thickness for $x \in [l_2, L]$; h_1 is the thickness for $x \in [0, l_1]$.
 $h(x)$ is the thickness for $x \in [l_1, l_2]$.

γ is defined as the thickness ratio and $\gamma = h_1/h_2$.



The mechanical model of the single taper-leaf spring. Fig(A-2)

Based on the model in the Figure (A-2), when the load F is applied at the free end, the same normal stress at any position for $x \in [l1, l2]$

$$\sigma_x = \frac{6Fx}{bh^2(x)}$$

From Equation above the normal stress σ_B of Section B for $x = l2$ can be expressed as

$$\sigma_B = \frac{6Fl_2}{bh_2^2}$$

In order to meet the requirements of the equal stress at Section B for $x = l2$, $\sigma_x = \sigma_B$ must be satisfied

$$h(x) = h_2 \sqrt{\frac{x}{l_2}} \quad h_1 \text{ can be expressed as: } \quad h_1 = h_2 \sqrt{\frac{l_1}{l_2}}$$

When the load F is applied at the free end, the deformation energy U can be expressed as:

$$U = \int_L \frac{(Fx)^2}{2EI} dx = \int_0^{l_1} \frac{(Fx)^2}{2EI_1} dx + \int_{l_1}^{l_2} \frac{(Fx)^2}{2EI_2} dx + \int_{l_2}^L \frac{(Fx)^2}{2EI_3} dx$$

Where I_1, I_2, I_3 are the inertia moments at different thicknesses and

$$I_1 = \frac{bh_1^3}{12}, I_2 = \frac{bh^3(x)}{12}, I_3 = \frac{bh_2^3}{12}$$

According to the Castigliano Second Theorem, the end deformation of the spring can be expressed as:

$$y = \frac{\partial U}{\partial F} = \frac{4F[L^3 + l_2^3(1 - \gamma^3)]}{Eb h_2^3}$$

Based on this Equation, the stiffness of the half of the single taper-leaf spring can be expressed as:

$$K_s = \frac{Eb h_2^3}{4[L^3 + l_2^3(1 - \gamma^3)]}$$

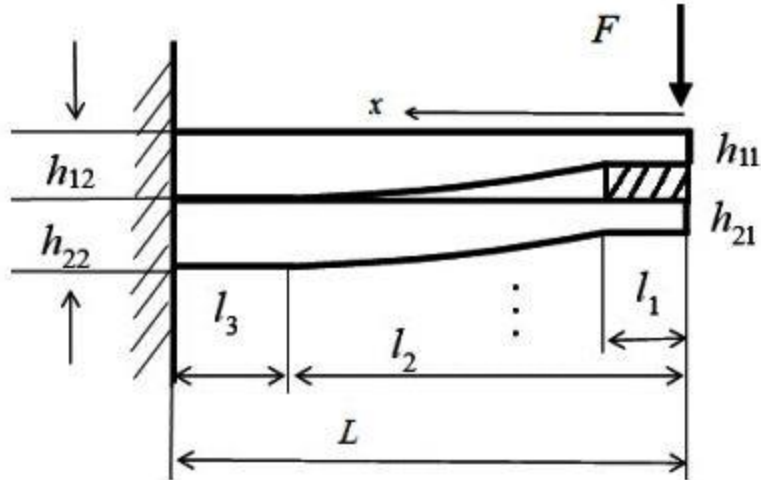
The design formula of the root thickness can be expressed as

$$h_2 = \sqrt[3]{\frac{4K_s[L^3 + l_2^3(1 - \gamma^3)]}{Eb}}$$

13. The Mechanical Model of the Taper-Leaf Spring Including n Pieces

The half of the taper-leaf spring including n pieces ($n = 2$ or 3) is taken as the research object. Its mechanical model is shown in Figure (3). Similarly, the half of the single taper-leaf spring also can be regarded as an elastic beam. It is fixed at one end. A concentrated load F is applied at the other end. The geometric parameters and the coordinate system are marked in Figure 3. The end displacement

of the taper-leaf spring is y . γ_i is defined as the thickness ratio and $\gamma_i = h_{i1}/h_{i2}$. Moreover, all the design values of γ_i are the same in practical engineering application and $\gamma_i = \gamma$.



The mechanical model of the taper-leaf spring including n pieces.

Figure (3)

$$y_i = y$$

where y_i the displacement of the i th piece at the free end ($i = 1, 2$ or $i = 1, 2, 3$).

y_i can be expressed as:

$$y = \frac{\partial U}{\partial F} = \frac{4F[L^3 + l_2^3(1 - \gamma^3)]}{Ebh_2^3} \Rightarrow y_i = \frac{4F_i[L^3 + l_2^3(1 - \gamma^3)]}{Ebh_{i2}^3}$$

Where F_i is the load shared by the i th piece

$$\frac{F_1}{h_{12}^3} = \frac{F_2}{h_{22}^3} = \dots = \frac{F_i}{h_{i2}^3} = \dots = \frac{F_n}{h_{n2}^3} = \frac{F}{h_{e2}^3}$$

The sum of the concentrated forces is equal to the concentrated load F , that is:

$$F_1 + F_2 + \dots + F_n = F$$

h_{e2} can be calculated by

$$h_{e2} = \sqrt[3]{h_{12}^3 + h_{22}^3 + \dots + h_{n2}^3}$$

F_i can be calculated by

$$F_i = \frac{h_{i2}^3}{h_{e2}^3} F$$

Under the load F, the maximum normal stress of the ith piece can be expressed as

$$\sigma_{imax} = \frac{6F_i L}{bh_{i2}^2} \Rightarrow \sigma_{imax} = \frac{6FLh_{i2}}{bh_{e2}^3}$$

So the designer or the engineer can design and adjust the leaf spring characteristics and performance based on the existing factors and the desired results.

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